$y = \Phi(x) \qquad \Phi : D \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$$\Phi(x) = \begin{bmatrix} \Phi_1(x) \\ \Phi_2(x) \\ \vdots \\ \Phi_m(x) \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\Phi(a,b) = a^2 - b^2 = (a+b)(a-b)$$

Alg1.

$$y = \underbrace{e^{\sin(x^2+1)}}$$

Alg2.

$$\begin{tikzcd} x \arrow[r] & \tilde{x} \arrow[r] \arrow[r, bend left, "\varepsilon_1"] & \tilde{x}^2 \arrow[r] \arrow[r, bend left, "\varepsilon_2"] & \tilde{x}^2+1 \arrow[r] \arrow[r, bend left, "\varepsilon_3"] & \sin(\tilde{x}^2+1) \arrow[d] \arrow[d, bend left, "\varepsilon_4"] \\ & & & & e^{\sin(\tilde{x}^2+1)} \end{tikzcd}$$

$$y = \Phi(x)$$

$$\Phi^{(i)} : D_i \longrightarrow D_{i+1}, \quad i = 0, 1, \dots, r, \quad D_i \subseteq \mathbb{R}^{n_i}$$

$$D_0 = D$$

$$\phi = \Phi^{(r)} \circ \Phi^{(r-1)} \circ \cdots \circ \Phi^{(1)} \circ \Phi^{(0)} \qquad D_{r+1} \subseteq \mathbb{R}^{n_{r+1}} = \mathbb{R}^m$$

توابع مقدماتی

$$x^{(0)} = x, \quad x^{(i)} = \begin{bmatrix} x^{(i)}_1 \\ x^{(i)}_2 \\ \vdots \\ x^{(i)}_{n_i} \end{bmatrix}, \quad \Phi^{(i)}\left(x^{(i)}\right) = x^{(i+1)} \in \mathbb{R}^{n_{i+1}}$$

$$x = x^{(0)} \longrightarrow \Phi^{(0)}\left(x^{(0)}\right) = x^{(1)} \longrightarrow \Phi^{(1)}\left(x^{(1)}\right) = x^{(2)} \longrightarrow \cdots$$

$$\longrightarrow \Phi^{(r)}\left(x^{(r)}\right) = x^{(r+1)} = y$$

$$\Phi(a,b,c) = a+b+c \in \mathbb{R}^1$$

$$\text{Alg1.}\quad \Phi^{(0)}(a,b,c) = \begin{pmatrix} a+b \\ c \end{pmatrix} \in \mathbb{R}^2, \quad \Phi^{(1)}(u,v) = u+v \in \mathbb{R}^1$$

$$\text{Alg2.}\quad \Phi^{(0)}(a,b,c) = \begin{pmatrix} a \\ b+c \end{pmatrix} \in \mathbb{R}^2, \quad \Phi^{(1)}(u,v) = u+v \in \mathbb{R}^1$$

$$\mathbb{R}^3 \longrightarrow \mathbb{R}^2 \longrightarrow \mathbb{R}^1$$

$$\Phi(x) = \begin{pmatrix} \overset{y_1}{\Phi_1(x_1,\dots,x_n)} \\ \vdots \\ \underset{y_m}{\Phi_m(x_1,\dots,x_n)} \end{pmatrix}, \quad \Phi^{(i)}\underset{\substack{\downarrow \\ \mathbb{R}^{n_i}}}{(u)} = \begin{pmatrix} \Phi_1^{(i)}(u) \\ \Phi_2^{(i)}(u) \\ \vdots \\ \Phi^{(i)}_{n_{i+1}}(u) \end{pmatrix}$$

$$x_i \longrightarrow \tilde{x}_i \qquad \Delta x_i = \tilde{x}_i - x_i \qquad \Delta y = \begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_m \end{bmatrix}$$

$$\tilde{y} = \Phi(\tilde{x}) \qquad \Delta y_i = \tilde{y}_i - y_i$$

$$\Delta y_i = \tilde{y}_i - y_i = \Phi_i(\tilde{x}) - \Phi_i(x) = \sum_{j=1}^{n} (\tilde{x}_j - x_j) \frac{\partial \Phi_i(x)}{\partial x_j} + \dots$$

$$\frac{\Delta y_i}{y_i} = \sum_{j=1}^{n} x_j \frac{\partial \Phi_i(x)}{\partial x_j} \Big/ y_i \cdot \frac{\Delta x_j}{x_j}$$

$$\underbrace{\begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_m \end{bmatrix}}_{\Delta y} = \begin{bmatrix} \frac{\partial \Phi_1}{\partial x_1} & \cdots & \frac{\partial \Phi_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \Phi_m}{\partial x_1} & \cdots & \frac{\partial \Phi_m}{\partial x_n} \end{bmatrix} \underbrace{\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}}_{\Delta x}$$

$$y_i = \Phi_i(x)$$

Jacobian matrix

ژاکوبین

$$\Delta y = D\Phi(x)\,\Delta x$$

$$\varepsilon_{y_i} = \sum_{j=1}^{n} \frac{x_j}{\Phi_i(x)} \cdot \frac{\partial \Phi_i(x)}{\partial x_j} \cdot \varepsilon_{x_j}$$

Condition number

$$y_i = \Phi_i(x_1, \dots, x_n)$$

$$x_1 \to \varepsilon_{x_1} \to \varepsilon_{y_i}, \qquad x_n \to \varepsilon_{x_n} \to \varepsilon_{y_i}$$

$y = f(x) \qquad f: \mathbb{R} \to \mathbb{R}$

$$\hookrightarrow \text{عدد حالت} = \frac{x f'(x)}{f(x)}$$

$$f(x) = \sin^{-1}(x) \longrightarrow \frac{x f'(x)}{f(x)} = \frac{x}{\underbrace{\sqrt{1-x^2}}_{\to 0} \, \underbrace{\sin^{-1}(x)}_{\to \frac{\pi}{2}}}$$

$x \approx 1$

$$y = \sqrt{x} \longrightarrow \frac{x f'(x)}{f(x)} = \frac{1}{2}$$

گشت مانده: $\quad \psi^{(i)} = \Phi^{(r)} \circ \Phi^{(r-1)} \circ \cdots \circ \Phi^{(i)} : D_i \longrightarrow \mathbb{R}^m$

$i = 0, 1, 2, \ldots, r$

$\psi^{(0)} = \Phi$

$$D(f\circ g)(x) = Df(g(x))\,Dg(x)$$

$$X = X^{(0)}$$

$$D\Phi(X^{(0)}) = D\Phi^{(r)}(X^{(r)})\,D\Phi^{(r-1)}(X^{(r-1)})\cdots D\Phi^{(0)}(X^{(0)})$$

$$D\psi^{(i)}(x^{(i)}) = D\Phi^{(r)}(x^{(r)})\,D\Phi^{(r-1)}(x^{(r-1)})\cdots D\Phi^{(i)}(x^{(i)})$$

$$i = 0, 1, \ldots, r$$